Coded OFDM With Transmitter Diversity for Digital Television Terrestrial Broadcasting (Corrected)*

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Abstract—Space-frequency coded (SFC) and channel coded orthogonal frequency-division multiplexing (COFDM) are considered under narrow-band interference (NBI). Analytical expressions for the bit error probability (BEP) are derived for OFDM with SFC in a frequency-selective fading environment. It is shown that SFC increases the resistance of COFDM against the NBI and reduces the BEP considerably. Specific attention is paid to Digital Terrestrial Television Broadcasting (DTTB), and the associated coding gains are discussed.

Index Terms—COFDM, digital video broadcasting, multitone interference, space-frequency coding.

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) is a multi-carrier modulation technique in which the available bandwidth is split among closely spaced, mutually orthogonal subcarriers. In recent years, OFDM-based technologies have been deployed in terrestrial digital video broadcasting (DVB-T) [10], wireless local area networks (LANs) such as HIPERLAN/2 [9], high bit rate digital subscriber line (HDSL) and asymmetric digital subscriber loop (ADSL) [2]. OFDM performs insufficiently in multipath fading channels with nulls, which results in very low signal-to-noise ratios (SNR) and a degradation in the quality of the communication. Channel coding is suggested to overcome the deep notches in the channel and coded OFDM (COFDM) technique has become popular in the past decade [11].

DVB-T is the European digital terrestrial broadcasting standard, which supplies 5 to 30 Mbps data rate on the 8 MHz ultra high frequency (UHF) channel [10]. Its performance has been analysed under several interference types such as impulsive noise [30], phase noise [6] and analog co-channel interference [26]. Besides these, in many applications such as DVB-T, COFDM systems are disturbed by narrow-band interference (NBI) in the channel, which causes an increment in the error probabilities [18], [19]. For instance, inappropriate spectrum allocation plans or inter-modulation products triggered by bad receiver designs can engender the OFDM system in DVB to be affected by narrow-band interference, which is modeled as multi-tone interference (MTI) [29]. Co-channel analog TV interference such as phase alternating line (PAL), is also a kind of tone variety interference [21], [23]. It is evident that there is a need for proper analytical tools to assess the performance of COFDM in the presence of interfering tones.

Although turning OFDM into COFDM makes the system more resistant to the various types of interference, appropriate measures need to be taken to ensure that interference does not hamper reliable high data rate communications. In [17], [20], [31] several techniques are developed to reduce the impact of narrow-band interference on OFDM systems. On the other hand, in [5], [13], [24] several antenna diversity techniques are proposed with applications to DVB, but none are for the purpose of interference mitigation. The use of OFDM offers the possibility of diversity in the frequency domain through space-frequency coding (SFC) [3]. While SFC is intended to provide gains against fading, it is also a proven technique to alleviate the negative impact of partial-band noise jamming in orthogonal frequency-division multiple access (OFDMA) [8] and multi-user interference in frequency-hopping OFDMA [14].

In this paper, we consider COFDM with SFC over two transmit antennas for DVB-T applications, and evaluate its performance in the presence of NBI in a frequency-selective fading environment. Space-frequency coding supplies interference-mitigation capability because there are instances where the interference may not affect both subcarriers that are used in SFC, and the post-decoding signal-to-interference plus noise (SINR) ratio is higher than that experienced by COFDM alone [7]. Analysis and simulations for various signal-to-noise (SNR) ratios and signal-to-interference ratios (SIRs) demonstrate the value added by SFC on top of its known advantages.

The organization of the paper is as follows. Section II briefly outlines the SFC-COFDM system, including the SINR ratio calculations. Hit probabilities for SFC-COFDM are computed in Section III. Bit error probability performances of uncoded OFDM and SFC-OFDM are furnished and an upper bound for coded system is derived in Section IV. The simulation model and results are in Section V. Conclusions are drawn in Section VI.

II. SPACE-FREQUENCY CODED OFDM

The block diagram of the COFDM system equipped with SFC capability is given in Fig. 1. Since our concentration is on inner coding/interleaving, the outer coding/interleaving part is omitted. After channel coding and interleaving, data symbols,
each with duration $T$, are modulated with quadrature phase shift keying (QPSK) or 16-point quadrature amplitude modulation (16-QAM), and fed into a serial-to-parallel converter to generate the $n$th OFDM block, which is denoted by $\mathbf{s}(n)$.

$$\mathbf{s}(n) = [S_0(n) \ S_1(n) \ldots \ S_{N-2}(n) \ S_{N-1}(n)]^T$$

where $S_m(n) := S(nN + m)$, $m = 0, 1, \ldots, N - 1$, is the $m$th symbol of the $n$th data block. Using the $n$th OFDM block $\mathbf{s}(n)$, the SF encoder generates two data vectors [15]

$$\mathbf{s}_1(n) = [S_0(n) - S_1(n) \ldots S_{N-2}(n) - S_{N-1}(n)]^T,$$

$$\mathbf{s}_2(n) = [S_1(n) \ S_0(n) \ldots S_{N-1}(n) - S_{N-2}(n)]^T.$$

We ignore the $(n)$ designation for notational simplicity.

Before transmission, inverse discrete Fourier transform (IDFT) is applied to both vectors and cyclic prefixes are appended. Subsequently, $\mathbf{s}_1$ and $\mathbf{s}_2$ are simultaneously transmitted over the respective antennas which are positioned such that the corresponding channels are uncorrelated. At the receiver, following the cyclic prefix removal, discrete Fourier transform (DFT) is applied. The DFTs of channel impulse responses $h_1 \equiv h_1(n)$ and $h_2 \equiv h_2(n)$ observed by transmit antennas, which are assumed to stay constant for the duration of the block, have the channel transfer functions $H_{1,m}, H_{1,n}, \ldots, H_{1,N-1}$ and $H_{2,0}, H_{2,1}, \ldots, H_{2,N-1}$, respectively, at the $N$ subcarriers.

Following OFDM demodulation, the received symbols for the subcarrier pair $(m, m+1)$ are

$$Y_m = H_{1,m}S_m + H_{2,m}S_{m+1} + Z_m + G_m \delta(m),$$

$$Y_{m+1} = -H^*_{1,m}S_{m+1} + H^*_{2,m}S_m + Z^*_m + G^*_m \delta(m + 1),$$

for $m = 0, 2, 4, \ldots, N - 2$, where $Z_m$ represents the zero-mean, complex, additive white Gaussian noise (AWGN) with two-sided power spectral density of $N_0/2$, $L_m$ and $G_m$ represent the NBI signal following OFDM demodulation, and the channel transfer function experienced by the interference, respectively. The indicator $\delta(m)$ is 1 if the $m$th subcarrier is interfered, and 0 if not.

Assuming the availability of perfect channel state information (CSI) at the receiver, the zero-forcing space-frequency decoder of [27] generates the decision estimates through

$$\hat{\mathbf{s}} = \mathbf{Cy}$$

(2)

where

$$\mathbf{C} = \begin{bmatrix}
H_{1,m}H^*_{1,m} + H_{2,m}H^*_{2,m} & 0 \\
\sqrt{H_{1,m}^2 + H_{2,m}^2} & 0 \\
H^*_{2,m+1} & H_{2,m+1} - H_{1,m}
\end{bmatrix}$$

(3)

$$\mathbf{s} = [\hat{S}_m \ \hat{S}_{m+1}]^T$$

and $\mathbf{y} = [Y_m \ Y_{m+1}]^T$. The zero-forcing decoder in (3) leaves the variance of noise or interference as they are in $\mathbf{y}$. From (2), the SFC decoder outputs are

$$\hat{S}_m = \frac{H_{1,m}H^*_{1,m+1} + H_{2,m}H^*_{2,m+1}}{\sqrt{H_{1,m}^2 + H_{2,m}^2}} S_m + Z_m,$$

$$\hat{S}_{m+1} = \frac{H^*_{1,m+1}S_{m+1} + H^*_{2,m}S_m}{\sqrt{H_{1,m}^2 + H_{2,m}^2}} Z_m + Z^*_{m+1}$$

(4)

where the post-decoding noise terms $Z_m$ and $Z^*_{m+1}$ are Gaussian with the same variance as $Z_m$ and $Z^*_{m+1}$, $m = 0, 2, 4, \ldots, N - 2$, for known CSI.

Suppose that the neighboring subcarriers have approximately the same channel transfer functions, i.e., $H_{1,m+1} \approx H_{1,m}$ and $H_{2,m+1} \approx H_{2,m+1}$. The validity of the above assumption is justified in [8]. Then, the SINRs at the decoder output when interference is absent (denoted by $\gamma^0$) are represented as

$$\gamma^0_m \approx \gamma^0_{m+1} \approx (|H_{1,m}|^2 + |H_{2,m}|^2) \frac{E_s}{N_0}$$

(6)

where $E_s$ is the symbol energy and $N_0$ is the one-sided noise power spectral density.

In this paper, we model NBI as tone interference, and consider two different NBI scenarios for the OFDM system.

**NBI-1:** The total NBI power $P_{tot}$ is distributed over $Q$ random, but not necessarily contiguous subbands. The received effective interference power spectral density in any subband is $P_{I} = P_{tot}/W_{tot}$, assuming that $P_{tot}$ is uniformly distributed over the entire bandwidth $W_{tot}$. This scenario represents the effect of intermodulation products to a certain extent, and gives the worst case performance of the system.

Let $\mathcal{I}(n)$ be the index set of subcarriers that are subject to NBI for OFDM block $n$. The interference source does not commit to specific subbands. It is assumed that the transmitter does not have prior information on $\mathcal{I}(n)$. If the transmitter has prior information on $\mathcal{I}(n)$, which is usually the case with co-channel interference, then NBI can be effectively eliminated by prohibiting those subcarriers in $\mathcal{I}(n)$ from the IDFT operation in the OFDM modulation process. However, this results in a waste of subcarriers, which can be avoided by employing SFC, as will be demonstrated in the sequel. Moreover, $\mathcal{I}(n)$ may vary in each

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OFDM block, which happens in hostile jamming, and rapid subcarrier excision may not be possible. The NBI, $I(t)$, is modeled as multiple tones such that

$$I(t) = \sum_{m \in \mathcal{I}(n)} \sqrt{\beta_m} e^{j(2\pi f_m t + \phi_m)}$$

(7)

where $\beta_m$, $f_m$ and $\phi_m$ are the power, the frequency and the phase of the NBI that hits the $m$th subcarrier, respectively. A sample distribution of the MTI is shown in Fig. 2(a), in which tone powers are assumed to be equal.

NBI-2: This case models adjacent channel interference that may be encountered at the two ends of the allocated broadcast spectrum. Analog TV interference (e.g., PAL) [21], [29] and DTBB to DTBB interference [21], [23], [29] fall in this category. The interference, which is again modeled as MTI, disturbs $Q$ subcarriers that reside permanently on the two edges of total transmission spectrum according to a certain power profile. The most powerful tones are placed at the first and the last subbands and tone powers start to decrease linearly towards the middle of the spectrum. The average received interference power in an interfered subband is $P_{\text{I}} = P_{\text{tot}}/Q$. A sample distribution of the NBI is shown in Fig. 2(b).

To consider the effects of NBI on the SINR at the SF decoder output, three distinct interference scenarios are examined in terms of the subcarrier pairs involved in SFC. (S1) None of the subcarriers is disturbed by the interference. (S2) A single subcarrier is disturbed by the interference. (S3) Both subcarriers are disturbed by the interference.

Throughout the rest of the analysis, SINR of the pair $m$ subject to a single interference and a double interference is represented by $\gamma_m^1$ and $\gamma_m^2$, respectively. The first scenario (S1) is already investigated in (6).

In (S2), if the $m$th carrier is interfered, $\delta(m) = 1$ and $\delta(m+1) = 0$ in (1); or else, if the $(m+1)$th carrier is interfered, $\delta(m) = 0$ and $\delta(m+1) = 1$ in (1) for $m = 0, 2, \ldots, N-2$. The SF decoder generates the decision estimates through (2) and therefore the SINR at the SF decoder output is found to be

$$\gamma_m^1 \approx \frac{1}{N_0 + |G_m|^2 \beta_m}$$

where $\beta_m = N/\beta_m/\text{tot}$. Finally, when both of the carriers are interfered, $\delta(m) = 1$ and $\delta(m+1) = 1$ in (1) and the corresponding SINRs at the decoder output become

$$\gamma_m^2 \approx \frac{1}{N_0 + |G_m|^2 \beta_m + |G_{m+1}|^2 \beta_{m+1}}.$$ 

III. Computation of the Tone-Interference Probabilities for NBI-1

Consider a SF coded system with two TX antennas so that there are $N/2$ carrier pairs in total. Of the $N$ subcarriers, $Q$ are disturbed by NBI, which is modeled as a collection of multiple noncontiguous tones, as described by (7). In NBI-1, each carrier pair is either subject to a double-interference, a single-interference or no interference. Because each symbol is sent on two distinct subcarriers, it is possible that a symbol interfered in one carrier will not be disturbed in the other one. A detailed derivation of the tone-interference probabilities can be found in [7] within the context of OFDMA.

A. Subcarrier Pairs Subject to a Double-Interference

The probability that at least $d \leq Q/2$ out of $N/2$ subcarrier pairs experience a double-interference with no constraint on the rest of the interferers (if there are any) is

$$P_D(d) = \frac{Q!}{2^d d!} \frac{(N/2)!}{(N-2k)!/((N-2k)/(N-2k-1))} \prod_{n=1}^{s} \frac{Q!/(2^d d!)}{s!} \frac{(N-2d+n-1)!}{(N-2d+n-1)!}.$$

where here and in the sequel it is assumed that $\prod_{k=0}^{d} f(k) = 1$.

The expected number of subcarrier pairs subject to a double-interference is

$$E[d] = \sum_{d=1}^{\lfloor Q/2 \rfloor} dP_D(d).$$
Finally, the probability that any subcarrier pair experiences a double-interference is

\[ P_2 = \frac{E[d]}{N/2}, \]

B. Subcarrier Pairs Subject to a Single-Interference

With a total of \( Q \) interferers, at least \( s \) out of \( N/2 \) subcarrier pairs experience single-interference with probability \( \prod_{n=0}^{s-1} \frac{(N - 2n)}{(N - n)} \), and the remaining \( Q - s \) tones contribute double-interference. Hence, \( \left(\frac{N/2-s}{d}\right) \prod_{k=0}^{s-1} \frac{(N - 2k - 1)}{(N - 2k)} \) is the probability of observing \( d = [(Q - s)/2] \) double-interference pairs given that there are \( s \) single-interference pairs. Recalling that there are \( Q!/(2^s s! \) interference scenarios, the probability that \( s \) pairs are subject to single-interference is

\[
P_S(s) = \frac{Q!}{s!d!} \frac{(N/2-s)}{d} \prod_{k=0}^{s-1} \frac{(N - s - 2k - 1)}{(N - s - 2k)} \times \frac{s}{N} \frac{N - 2n}{N - n},
\]

The expected number of subcarrier pairs with a single-interference is

\[ E_s = \sum_{s=1}^{\min(N/2,Q)} sP_S(s), \]

and consequently, any given subcarrier pair experiences a single-interference with probability

\[ P_1 = \frac{E_s}{N/2}, \]

IV. Bit Error Probability Performance Under MT Interference

A. Uncoded OFDM

1) QPSK: The average BEP of QPSK modulation in an AWGN channel is

\[ P_b_{\text{QPSK}}(\gamma_s) = Q(\sqrt{\gamma_s}) \]

where \( \gamma_s = E_b/N_0 \), and \( Q(x) = \left(1/\sqrt{2\pi}\right) \int_x^\infty e^{-u^2/2} du \). The NBI disturbs \( Q \) subcarriers among \( N \) possible subcarriers with a total power \( P_{\text{tot}} \). The BEP of the QPSK symbol becomes

\[ P_b_{\text{QPSK}} = \frac{1}{2\pi} \int_0^{2\pi} Q \left( \sqrt{\frac{2E_b}{N_0} \left[ 1 - \sqrt{\frac{3m}{E_b}} \sin \theta \right]} \right) d\theta \]

in the presence of both tone interference and AWGN, where \( E_b \) is the average energy per bit [25].

The BEP of the OFDM system averaged over all subcarriers can be written as

\[ P_b_{\text{OFDM}} = \frac{1}{N} \sum_{m=1}^N P_b(\gamma_m), \]

where \( P_b(\gamma_m) \) represents the bit error probability of the \( m \)th carrier with the instantaneous SINR \( \gamma_m \). For NBI-2, when \( Q \) of the total \( N \) subcarriers are subject to interference, inserting (10) and (11) into (12) gives

\[ P_b_{\text{OFDM}} = \frac{1}{N} \sum_{m=1}^N \left[ \frac{H_{1,m}^2 E_b}{N_0} \right] \left[ \frac{2\beta_m}{N_0} \sin \theta \right] \]

and \( \gamma_m \) are the index sets of subcarriers with and without interference, respectively. The two sets have the cardinalities \( |\mathcal{T}| = Q \) and \( |\mathcal{T}'| = N - Q \). On the other hand, the BEP becomes

\[ P_b_{\text{OFDM}} = \frac{1}{N} \left( \frac{N - Q}{Q} \right) \left( \frac{H_{1,m}^2 E_b}{N_0} \right) \left[ \frac{2\beta_m}{N_0} \sin \theta \right] \]

for NBI-2, where \( \mathcal{M} = 2P_1Q^2/(Q(Q + 2)) \), and \( \beta_m = P_1N/Q \) is the average power spectral density of tones in interfered subcarriers. The term \( \mathcal{M} = 2P_1Q^2/(Q(Q + 2)) \) introduces the degradation of tone powers towards the middle of the spectrum (e.g., for \( Q = 8 \), \( \mathcal{M} = 8P_1/5 \), and thus the power spectral densities of the tones in their respective bands is as follows starting with the first subcarrier: \( 8P_1/5, 6P_1/5, 5P_1, 5P_1/2, \ldots, 0 \)).

For simplification, the following assumption is made in the ensuing calculations.

(A1) The interference signal does not experience a severe fading (e.g., the interferer is close to the transmitter), and therefore \( |G_m| = 1 \).
realistic, (A1) represents a worst case scenario from the transmis-
mitter’s perspective.

Considering Rayleigh fading, the expected BEP is

\[
P_b^{OFDM} = \int_0^\infty \cdots \int_0^\infty p_b^{(2)}(\gamma) d\gamma
\]

(13)

for \(\gamma = [\gamma_0 \quad \gamma_1 \quad \cdots \quad \gamma_{N-1}]^T\) where \(\gamma_m = |H_{1,m}|^2 E_s/N_0\) is chi-squared with two degrees of freedom (DOF) and \(p(\gamma)\) is the multivariate chi-squared probability density function (pdf) that represents the frequency-selective channel.

2) 16-QAM: The average BEP of 16-QAM modulation under AWGN is [25]

\[
P_{b,16-QAM}(\gamma_S) = \frac{3}{4} Q(\sqrt{\gamma_S/5}) + \frac{1}{2} Q(3\sqrt{\gamma_S/5}) - \frac{1}{4} Q(5\sqrt{\gamma_S/5}),
\]

whereas the average BEP of 16-QAM modulated symbols under both AWGN and NBI is [25]

\[
P_{b,16-QAM} = \frac{1}{4\pi} \int_0^{2\pi} [P_c(\theta) + P_s(\theta)] d\theta
\]

where

\[
P_c(\theta) = Q \left( \sqrt{\frac{E_s}{5N_0}} \left( 1 + \sqrt{\frac{10P_I}{E_s}} \cos \theta \right) \right)
\]

\[
+ \frac{1}{2} Q \left( \sqrt{\frac{E_s}{5N_0}} \left( 1 - \sqrt{\frac{10P_I}{E_s}} \cos \theta \right) \right),
\]

\[
P_s(\theta) = Q \left( \sqrt{\frac{E_s}{5N_0}} \left( 3 + \sqrt{\frac{10P_I}{E_s}} \cos \theta \right) \right)
\]

\[
- \frac{1}{2} Q \left( \sqrt{\frac{E_s}{5N_0}} \left( 5 + \sqrt{\frac{10P_I}{E_s}} \cos \theta \right) \right).
\]

Similar as the QPSK case, when \(Q\) of the total \(N\) subcarrier is subject to interference, the BEP of OFDM system with 16-QAM modulation is expressed as (14), shown at the bottom of the page, for NBI-1 and (15), also shown at the bottom of the page, for NBI-2.

By (A1), we can simplify (14) and (15), and then express the expected BEP under Rayleigh fading channel through (13).

B. Uncoded SFC-OFDM

While deriving the bit error probability of the SFC-OFDM system, subcarrier pairs should be taken into account. Thus, (12) can be modified as

\[
P_b^{SFC-OFDM} = \frac{1}{N} \left( P_b^1 + P_b^2 + \cdots + P_b^{N-2} \right),
\]

(16)
where \( P_b^m = P_b(\gamma_m) + P_b(\gamma_{m+1}) \) represents the total bit error probability of the contiguous subcarriers of the \( m \)th pair, \( m = 0, 2, \ldots, N - 2 \). The SINR changes according to the effect of interference on the subcarrier pairs. Hence, \( \gamma_m \) can be classified into three types: (1) \( \gamma_m^0 \) representing the interference-free SINR; (2) \( \gamma_m^1 \) representing the single-interference SINR; (3) \( \gamma_m^2 \) representing the double-interference SINR.

1) NBI-I:

\[ a) \text{QPSK: Defining } P_b(\gamma_i^0), i = 0, 1, 2, \text{as the probability of bit error for the given SINR in the } i \text{th carrier, } P_b^m \text{ can be expressed as the summation of different interference cases (e.g., double-interference, single-interference, interference-free)} ]

\[ P_b^m = [P_2 P_2 (\gamma_m^0) + P_1 P_1 (\gamma_m^1) + P_0 P_0 (\gamma_m^0)] + [P_2 P_1 (\gamma_{m+1}^0) + P_1 P_1 (\gamma_{m+1}^1) + P_0 P_0 (\gamma_{m+1}^0)]. \]

(17)

where \( P_1 \) and \( P_2 \) are defined in Section III, and \( P_0 = 1 - P_1 - P_2 \). Therefore, \( P_b(\gamma_m^0) \approx P_b(\gamma_{m+1}^0) \) can be found by substituting (6) into (10) as

\[ P_b(\gamma_m^0) \approx Q \left( \sqrt{\frac{[H_{1,m}^2 + H_{2,m}^2]}{N_0}} \right). \]

(18)

The probabilities \( P_b(\gamma_m^0) \) and \( P_b(\gamma_{m+1}^0) \) can be found in a similar fashion by substituting (8) into (11) as

\[ P_b(\gamma_m^0) \approx \frac{1}{2\pi} \int_0^{2\pi} Q \left( \sqrt{\frac{[H_{1,m}^2 + H_{2,m}^2]}{N_0}} \right) d\theta, \]

(19)

which is also equal to \( P_b(\gamma_{m+1}^0) \).

Next, we stipulate the following constraint on the interferer power distribution:

(A2) The adjacent subcarriers are subject to double-interference with about equal power, i.e., \( \beta_m \approx \beta_{m+1} \).

The assumption (A2) is representative of NBI that originates from adjacent bands in DTTB, for instance.

Invoking (A2), lastly, \( P_b(\gamma_{m}^2) \) and \( P_b(\gamma_{m+1}^2) \) are expressed by substituting (9) into (11) as

\[ P_b(\gamma_m^2) \approx \frac{1}{2\pi} \int_0^{2\pi} \left[ \sqrt{\frac{[H_{1,m}^2 + H_{2,m}^2]}{N_0}} \right] d\theta \]

(20)

which is the same as \( P_b(\gamma_{m+1}^2) \).

The average bit error probability of SFC-OFDM over a Rayleigh fading channel is

\[ P_b^{\text{SFC-OFDM}} = \prod_{i=1}^\infty P_b^{\text{SFC-OFDM}}(\gamma_i^0) \prod_{i=1}^\infty P_b^{\text{SFC-OFDM}}(\gamma_i^2) d\gamma_i^0 d\gamma_i^2. \]

(21)

where \( \gamma_i = [\gamma_i^0, \gamma_i^1, \ldots, \gamma_i, N-1]^T, i = 1, 2, \gamma_i^m = [H_{1,m} E_n / N_0, H_{2,m} E_n / N_0] \) is chi-squared with two DOF and \( p(\gamma_i^0), p(\gamma_i^2) \) are the multivariate pdfs of the channels experienced by the two transmit antennas. Inserting (16) in (21), the integral can be taken separately for each \( P_b^m \). Focusing on the integral of \( P_b^0 \), we assume the following.

(A3) Both channels experienced by each antenna are uncorrelated with each other, and adjacent subcarrier pairs experience uncorrelated channel gains.

Omitting the correlation between the pairs as per (A3) produces actually a lower bound for the BEP of the OFDM system. Nevertheless, (A3) simplifies the analysis and still helps explore the impact of interference on the performance.

Combining (16), (17), (21) and simplifying

\[ P_b^0 = \frac{1}{N} \int_0^\infty \int_0^\infty \int_0^\infty \left[ P_2 \left( P_b(\gamma_i^0) + P_b(\gamma_i^2) \right) \right. \]

\[ + P_1 \left( P_b(\gamma_i^0) + P_b(\gamma_i^1) \right) + P_0 \left( P_b(\gamma_i^0) + P_b(\gamma_i^1) \right) \]

\[ \times p(\gamma_i^0, \gamma_i^1) p(\gamma_i^2, \gamma_i^3) d\gamma_i^0 d\gamma_i^1 d\gamma_i^2 d\gamma_i^3 \]

(22)

where \( p(\gamma_i^0, \gamma_i^1, \gamma_i^2, \gamma_i^3) \), \( i = 1, 2 \), is the bivariate pdf for the adjacent subcarriers on antenna \( i \).

Since the sum of two chi-squared random variables with two degrees of freedom is chi-squared with four degrees of freedom, \( \gamma = \gamma_i + \gamma_2 \) has the pdf \( p(\gamma) = \gamma e^{-\gamma} / \gamma_0^2 \). Then, (22) leads to the following expression by substituting (18), (19), (20) into (22) and rearranging

\[ P_b^0 = \frac{1}{\pi} \left[ P_2 \int_0^\infty \int_0^{2\pi} Q \left( \sqrt{\frac{4/\gamma_0}{N_0}} \sin \theta \right) p(\gamma) d\gamma d\theta \right. \]

(23)

\[ + P_1 \int_0^\infty \int_0^{2\pi} Q \left( \sqrt{\frac{2/\gamma_0}{N_0}} \sin \theta \right) p(\gamma) d\gamma d\theta \]

\[ + 2\pi P_0 \int_0^\infty \int_0^{2\pi} \left( Q(\sqrt{\gamma}) p(\gamma) d\gamma \right). \]

These integrals can be numerically evaluated by using the trapezoidal integral evaluation method and the 180-point Gauss-Laguerre integration formula

\[ \int_0^\infty f(x) dx \approx \sum_{i=1}^{180} f(x_i) w_i e^{x_i}, \]

(23)
where \( x_i \) and \( w_i \) represent the abcissas and weights of the Gauss-Laguerre formula, respectively, and are obtained from the roots of the Laguerre polynomials [1], [4].

Finally, if \( \mathcal{P}_b^m \), \( m = 2, 4, \ldots, N - 2 \), are derived in a similar fashion as \( \mathcal{P}_b^0 \), the resulting average bit error probability of uncoded SFC-OFDM system over Rayleigh fading is

\[
\mathcal{P}_{b,\text{SFC-OFDM}} = \frac{N}{2} \left( \mathcal{P}_b^0 + \mathcal{P}_b^2 + \cdots + \mathcal{P}_b^{N-2} \right). \tag{24}
\]

\( \mathcal{P}_b^0 \)

b) 16-QAM: Following the same steps as those for QPSK case, \( \mathcal{P}_b^0 \) for 16-QAM is found to be (25), shown at the bottom of the page. Equation (25) can be numerically evaluated by the trapezoidal integration formula and (23). Finally, the average bit error probability of uncoded SFC-OFDM system with 16-QAM modulation is obtained through (24).

2) NBI-2:

a) QPSK: The average bit error probability of SFC-OFDM system with QPSK modulation under NBI is

\[
\mathcal{P}_{b,\text{QPSK}} = \frac{1}{N} \left( \sum_{i=1,2,\ldots,Q/2-1} \left( \int_0^{2\pi} \int_0^{\infty} \left( \sqrt{\gamma} - \sqrt{\frac{2\mathcal{T}_i}{N_0}} \sin \theta \right) p_\gamma(\gamma) d\gamma \right) \right) + \frac{1}{N} \left( \int_0^{\infty} \left( \frac{3}{4} Q\left( \sqrt{\gamma} / 5 + \frac{2\mathcal{T}_i}{N_0} \cos \theta \right) - \frac{1}{2} Q\left( \sqrt{\gamma} / 5 \right) \right) d\gamma \right).
\]

\( \mathcal{P}_b^0 \)

\( N = 2, 4, \ldots, N - 2 \)

\( Q(\cdot) \)

\( \mathcal{T}_i \)

\( N_0 \)

\( \mathcal{P}_b^0 \)

C. SFC-COFDM

It is known that channel coding augments the performance against interference and jamming. Convolutional coding can be considered for error-control purpose. For the BEP of SFC-COFDM, the following bound will be used with the assumption that the channel error probabilities are low enough [16]:

\[
P_b(E) < \frac{1}{k} \sum_{d=d_{\text{free}}} \infty B_d P_d \tag{28}
\]

where \( k \) is the number of information bits per unit time, \( d_{\text{free}} \) is the minimum weight codeword of any length produced by a nonzero information sequence, \( B_d \) is the total number of nonzero information bits on all weight \( d_{\text{free}} \) paths, and \( P_d \) is the event error probability.

Without channel coding, the diversity level of our uncoded system is equal to 2 which is equal to the multiplication of the number of transmit and receive antennas. But with channel coding, the diversity level increased to \( 2d \) [12]. Hence, while taking the average over Rayleigh fading channel the pdf of the chi-squared random variable will have \( 4d \) DOF instead of 4.
TABLE I
DVB-T SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>2k, Non-hierarchical</td>
</tr>
<tr>
<td>Number of subcarriers N</td>
<td>1705</td>
</tr>
<tr>
<td>Useful symbol duration</td>
<td>224 μs</td>
</tr>
<tr>
<td>Cyclic prefix duration</td>
<td>7 μs</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK, 16-QAM</td>
</tr>
<tr>
<td>Convolutional Encoder</td>
<td>Rate-1/2, (133,171) code</td>
</tr>
<tr>
<td>Interleaver</td>
<td>Bitwise block interleaver</td>
</tr>
</tbody>
</table>

Thus, $P_d$ can be found through (24), (25), (26) or (27), recalling that the average is taken over the chi-squared pdf with $4d$.

V. NUMERICAL EVALUATIONS AND SIMULATIONS

The SF block coding scheme shown in Fig. 1 is adopted and the performance of COFDM system with and without SFC is tested in the presence of tone interference. Simulation parameters are given in Table I. An additional subcarrier is included to make the total number even at 1706. Alternatively, one of the subcarriers may remain unoccupied in practice.

Symbols are transmitted over frequency-selective channels modelled as finite impulse response (FIR) systems that are of order five with complex Gaussian-distributed coefficients, and flat power delay profiles. The data symbols are also contaminated by zero-mean, complex AWGN. The channels experienced by each antenna are assumed to be uncorrelated and CSI is perfectly estimated at the receiver. Throughout the simulations of NBI-1, it is assumed that each interfering tone has the same power $P_I$ and the set $\mathcal{I}$ randomly changes at every OFDM block. The bit error rate (BER) results reflect the averages of 1,000 and 100 independent channel pair realizations for the uncoded cases and the coded cases, respectively. We define the signal-to-noise ratio as $\text{SNR} = \frac{E_b}{N_0}$, and the signal-to-interference ratio as $\text{SIR} = \frac{E_b}{P_I}$.

Figs. 3 and 4 show the uncoded BER performance of SFC-OFDM against SNR for NBI-1. They depict that at high SNRs, the performance limiting factor is the NBI. At low SIR such as 5 dB, diversity gain cannot be observed, but only a small performance improvement is attained with SFC. At high SIR and SNR, the diversity gain offered by SFC becomes noticeable. For achieving a BER of $1 \times 10^{-2}$ at $\text{SIR} = 20$ dB, the OFDM system with SFC requires about 6 dB less energy to combat noise and NBI compared to OFDM alone.

The BER performance against SIR for $\text{SNR} = 20$ dB is shown in Figs. 5 and 6. As SIR increases, the BER performance is augmented and SFC supplies a 7 and 8 dB SIR improvement.
for QPSK and 16-QAM cases, respectively. It should be noted that at low SNRs double-hit error probabilities increase enormously, and therefore SFC-OFDM performs slightly worse than OFDM. The expected bit error probability computation for SFC-OFDM is very close to the results obtained from averaging over channel realizations and simulations. This supports the assumption that adjacent subcarriers have almost identical channel transfer functions.

Figs. 7 and 8 show the uncoded BEP performance of SFC-OFDM against SNR and SIR for NBI-2. Similar as scenario 1, at high SNRs, the performance limiting factor is the NBI. In Fig. 7, for a BEP of $1 \times 10^{-2}$, the OFDM system with SFC requires about 6 dB less energy than OFDM does alone at SNR = 20 dB.

In the following figures, the effect of channel coding is investigated for various SNR and SIR values. The convolutional encoder is rate-1/2 with generator polynomials (133,171) and constraint length-7 in compliance with the DVB-T specifications. In calculating the bound in (28), the parameters are $B(d) = 36d^{10} + 211d^{12} + 140d^{14} + \ldots$ and $d_{\text{free}} = 10$ [16], [28].

Figs. 9 and 10 show the BEP performance of SFC-COFDM against SNR for NBI-1. Fig. 9 depicts that SFC offers about 8-dB SNR improvement at a BEP of $1 \times 10^{-2}$ with respect to COFDM with QPSK modulation for both NBI-1. In Fig. 10, as SNR increases the BEP performance of channel coding is augmented and SFC supplies a 4 dB SNR improvement at a BEP of $1 \times 10^{-2}$ for 16-QAM modulation. The SFC combined with channel coding reduces the BEP of the OFDM system about a 1.5 orders of magnitude at SNR = 15 dB. It can be concluded that interference mitigation of the SFC-OFDM system is improved significantly by channel coding.

Figs. 11 and 12 represent the BEP performance of SFC-COFDM against SIR for NBI-1 and NBI-2. Fig. 11 depicts that space-frequency coding combined with channel coding reduces the BEP of the OFDM system about two orders of magnitude at a SIR of 15 dB for 16-QAM. Similarly, SFC offers about 9-dB SNR improvement at a BEP of $5 \times 10^{-3}$ with respect to COFDM in Fig. 12. The BEP upper bound (28) is also presented in the figures, and the simulation results agree with it. Unfortunately, the bound gives inappropriate results for low SNR or SIR, and in some figures it even diverges. Although the upper bound is loose, it becomes stronger as the interference and noise powers decrease. As the interference power drops, the contribution of SFC to the robustness against interference improves.

When the performances of COFDM and SFC-COFDM systems under NBI-1 and NBI-2 are compared, it is observed that BEPs under NBI-2 are lower than the ones under NBI-1 for all SIR values. Because the tone powers are linearly decreasing in
Fig. 9. BEP versus SNR for SFC-COFDM in NBI and frequency-selective fading (QPSK, SIR = 7 dB, Q/\sqrt{N} = 0.25, NBI-1).

NBI-2, the tones with power less than \( P_f \) may not seriously disturb the carriers, or in other words, channel coding may be more effective on these carriers. On the other hand, the bound in Fig. 12 is loose for almost all SIR values. While computing the bound, the subcarriers on the edges of the bandwidth, which are subject to high tone powers, cause very high error probabilities and this prevents the bound from giving tight results.

VI. CONCLUSION

We have investigated the possible benefits of employing space-frequency coding with zero-forcing decoding against MT interference in a COFDM system for DTTB applications. Bit error probability analysis shows that SFC injects particularly significant gains in the medium-to-high SIR range, which is consistent with the behavior of transmit diversity. The influence of SFC on the BEP is related with the total interference power and \( Q \), and the reduction in the BEP relative to OFDM increases inversely proportional to SIR. The simulations of the SFC-COFDM system indicates that channel coding improves the performance of SFC against NBI, and SFC is a suitable diversity technique for DTTB applications. Considering the reduction observed at the BEP, higher order modulation or coding rates can be used to support higher data rates reliably.
REFERENCES


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