Analysis of Target Detection Probability in Randomly Deployed Sensor Networks

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***Abstract*— The main functionality of a surveillance wireless sensor network is to detect unauthorized traversals in a field. In this paper, we develop a formulation to determine the probability of detecting a randomly positioned target by a set of binary sensors to serve as the deployment quality measure. This formulation leads to a recipe to determine the number of sensors required to deliver a certain deployment quality level. The sensing- and communication-neighbor degrees which can be used as design criteria in a sensor network are defined and calculated. The model is verified by simulations whose outcomes closely match the analytical results.**

***Index Terms*— Sensor networks, deployment quality, probability of detection, sensing range.**

I. Introduction

CONSIDER a scenario where a set of sensor nodes are deployed randomly to a region to detect unauthorized traversals. Nodes are equipped with low-power wake-up radios, and the first detecting sensor alerts the others within the communication range. Sensors remain active for tracking purposes once a positive detection occurs. The following questions arise regarding the coverage performance: What is the quality of the deployment? Is the deployed number of sensors adequate to provide the required quality in terms of breach detection? As a first attempt to address these questions, experimental methods were proposed based on different deployment quality measures in [3]. In this paper, we present an analytical study to determine the required number of sensors based on the sensing coverage only. To that end, the probability of detecting a randomly positioned target by a set of binary sensors, which are adopted in [2] and other research work, is formulated. Using this formulation, it is possible to determine the expected number of sensors required to cover any randomly chosen point in the field.

We define three problems which are related to each other, and based on the following assumptions:

1) The field is rectangular.
2) The positions of the sensors are uniformly random.
3) The x and y coordinates are independent.
4) The deployed sensors are identical.
5) Binary detectors with sensing range d_t are utilized.
6) The communication range d_c of the sensors is at least twice the sensing range, $d_c \geqslant 2d_t$ [5].

The sensing-neighbor degree, w is defined as the number of sensors within the sensing range d_t of each other. Given d_{ij}, the distance between sensors i and j, define adjacency as $a_{ij} = 1$, if $d_{ij} \leqslant d_t$ and $a_{ij} = 0$, otherwise. Then, the sensing-neighbor degree of the ith sensor is $w_i = \sum_{i \neq j} a_{ij}$. The communication-neighbor degree, v is likewise defined as the number of sensors within the communication range d_c of each other. Redefine adjacency as $b_{ij} = 1$ if $d_{ij} \leqslant d_c$ and $b_{ij} = 0$, otherwise, the communication-neighbor degree of the ith sensor is $v_i = \sum_{i \neq j} b_{ij}$. The problems can now be stated as:

(Q1) What is the probability of detecting a randomly located target by at least one sensor given that N sensors are deployed? The solution to this problem can be considered as a deployment quality measure (DQM).

(Q2) What is the required number of sensors to meet a specified DQM value?

(Q3) What are the average sensing- and communication-neighbor degrees given that the communication range is at least twice the sensing range $d_c \geqslant 2d_t$?

II. Analytical Model

The target position is unknown. Suppose also that the positions of a set of sensors are uniform randomly distributed in a rectangular field where the length and the width are D_1 and D_2, respectively, with $D_1 \leqslant D_2$. The distance d between two random points, which represent the target and sensor positions, in the rectangular field is also as a random variable whose cumulative distribution and probability density functions are respectively defined as [4]

$$F(\xi D_1) = \begin{cases} 0, & \xi < 0 \\ \zeta\xi^2(\frac{\zeta\xi^2}{2} - \frac{4}{3}\xi(1+\zeta) + \pi, & 0 \leqslant \xi < 1, \\ \frac{2}{3}\zeta\sqrt{\xi^2 - 1}(2\xi^2 + 1) & \\ -\frac{1}{6}\zeta(8\xi^3 + 6\zeta\xi^2 - \zeta) & \\ +2\zeta\xi^2 \sin^{-1}(1/\xi), & 1 \leqslant \xi < \zeta^{-1}, \\ \frac{2}{3}\zeta\sqrt{\xi^2 - 1}(2\xi^2 + 1) & \\ -\frac{1}{2}\zeta^2(\xi^4 + 2\xi^2 - \frac{1}{3}) & \\ +\frac{2}{3}\sqrt{\xi^2 - \zeta^{-2}}(2\zeta^2\xi^3 + 1) & \\ +\frac{1}{6}\zeta^{-2} - \xi^2 & \\ +2\zeta\xi^2 \sin^{-1} 1/\xi & \\ -2\zeta\xi^2 \cos^{-1} 1/\zeta\xi, & \zeta^{-1} \leqslant \xi < \sqrt{1 + \zeta^{-2}} \\ 1, & \sqrt{1 + \zeta^{-2}} \leqslant \xi. \end{cases} \quad (1)$$

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$$f(\xi D_1) = \frac{1}{D_1} \begin{cases} 2\zeta^2\xi^3 + 2\zeta\xi\pi \\ -4\zeta\xi^2(1+\zeta), & 0 \leqslant \xi < 1, \\ 4\zeta\xi\sqrt{\xi^2-1} \\ -2\zeta\xi(2\xi+\zeta) \\ +4\zeta\xi\sin^{-1}(1/\xi), & 1 \leqslant \xi < \zeta^{-1}, \\ 4\zeta\xi\sqrt{\xi^2-1} \\ +4\zeta^2\xi\sqrt{\xi^2-\zeta^{-2}} \\ -2\xi(\zeta^2\xi^2+1+\zeta^2) \\ +4\zeta\xi\sin^{-1}(1/\xi) \\ -4\zeta\xi\cos^{-1}(\frac{1}{\zeta\xi}), & \zeta^{-1} \leqslant \xi < \sqrt{1+\zeta^{-2}}, \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

where $\zeta = D_1/D_2 \leqslant 1$ is the shape parameter and $\xi = d/D_1$. A binary detector can be formulated as

$$p(x) = \begin{cases} 1, & x \leqslant d_t, \\ 0, & x > d_t \end{cases} \tag{3}$$

where $x \geqslant 0$ is the sensor-to-target distance, d_t is the sensing range and $p(x)$ is the detection probability for each sensor. The binary model provides a very good approximation when the detection probability falls quite sharply, as is the case in passive infrared sensors for instance [1].

The probability, $\mathcal{P}$, of detecting a target at some unknown location by randomly deployed sensor is

$$\mathcal{P} = \int_0^\infty p(x)f(x)dx = \int_0^{d_t} f(x)dx = F(d_t). \tag{4}$$

Each sensing event can be considered as a Bernoulli trial with success probability $\mathcal{P}$ as follows:

- Choose two random positions in the rectangle.
- Assume that the target is located on one of the points and there is a sensor on the other.
- If the distance between these two random points is less than d_t, then the target is detected successfully, and the trial fails otherwise.

Repeating Bernoulli trials N times produces a binomial distribution. If N sensors are deployed, then the probability that a randomly positioned target is detected by k out of N sensors follows this binomial distribution. Hence,

$$p(N,k) = \binom{N}{k} F(d_t)^k [1-F(d_t)]^{N-k}. \tag{5}$$

Equation (5) cannot be approximated by a Poisson distribution because $\lim_{N\to\infty} NF(d_t) \to \infty$. However, because $NF(d_t)$ is large enough, (5) can be approximated by the normal distribution $\mathbb{N}(\mu,\sigma)$ where $\mu = NF(d_t)$ and $\sigma^2 = NF(d_t)[1-F(d_t)]$. The probability that the target is detected by at least one sensor is

$$\mathcal{P}_> = 1 - p(N,0) = 1 - [1-F(d_t)]^N, \tag{6}$$

and $\mathcal{P}_>$ is also the answer to (Q1).

Suppose that a preset DQM value is specified, i.e., the target detection probability $\mathcal{P}_>$ must be greater than or equal to some threshold $\mathcal{P}_t$. Using (6), one can find the minimum number of sensors, N^*, that will satisfy the $\mathcal{P}_> \geq \mathcal{P}_t$ requirement as

$$N^* = \left\lceil \frac{\log(1-\mathcal{P}_t)}{\log(1-F(d_t))} \right\rceil, \tag{7}$$

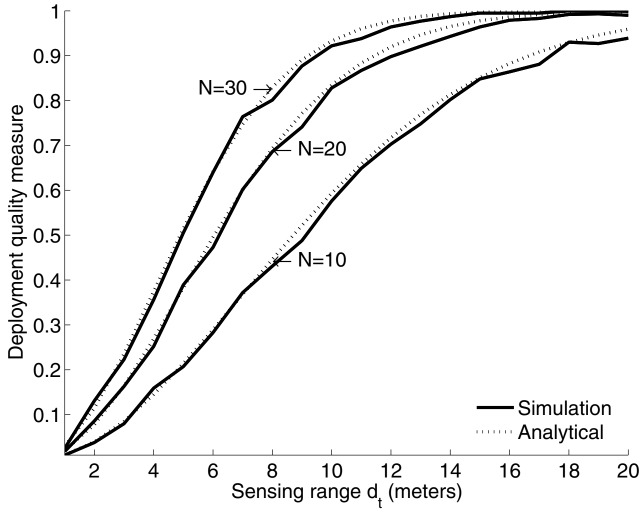

Fig. 1. Effect of sensing range on the DQM for $D_1 = 30$ m, $D_2 = 100$ m and $N = 10, 20, 30$.

which answers (Q2).

Concentrating on some particular sensor, we can define a Bernoulli trial as whether any other randomly deployed sensor is in the sensing range or not. If N sensors are deployed, then the sensing-neighbor degree of a sensor is the expected value of the binomial random variable obtained through $N-1$ Bernoulli trials. Consequently, the sensing-neighbor degree is

$$w = (N-1)F(d_t). \tag{8}$$

Because the communication range d_c is at least $2d_t$, we can redefine the trial according to the communication range, and find the communication-neighbor degree as

$$v \geqslant (N-1)F(2d_t). \tag{9}$$

The solutions in (8) and (9) address (Q3). Note that since $F(2d_t) > F(d_t)$, if the network is designed according to the communication range, breach holes will occur in the sensing coverage. In the next section, we present numerical evaluation of the proposed analytical deployment quality measure.

III. Numerical Evaluation

The simulations are performed with Matlab. The field of interest is modeled as a rectangle, and a set of sensors are deployed where the coordinates are generated according to a uniform distribution. In each run, a target is assumed to be located on a random point in the rectangular field. If the target is in the sensing range of any sensor, that is, if the distance between the target and any sensor is smaller than the sensing range, then the run is assumed to be a success, and it is a failure otherwise. For each deployment, the experiments are repeated 1000 times and the results are the averages of 1000 distinct deployments with different seeds. The effect of the number of sensors and the detector range on $\mathcal{P}_>$ are shown in Fig. 1 and Fig. 2, respectively. The variances in the simulations are in the acceptable range and verify the correctness of the analytical model.

As the sensing range increases, so does $\mathcal{P}_>$ as per (6). This is depicted in Fig. 1 for $N = 10, 20, 30$, where the DQM is plotted against the sensing range. As the number of deployed

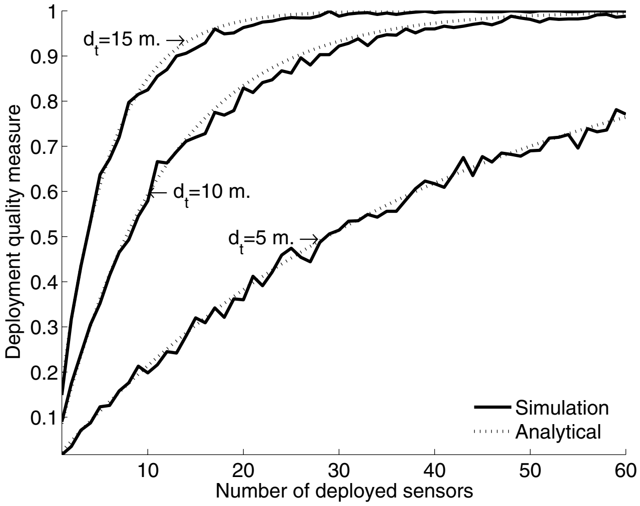

Fig. 2. Effect of number of sensors on the DQM for $D_1 = 30$ m, $D_2 = 100$ m and $d_t = 5, 10, 15$ m.

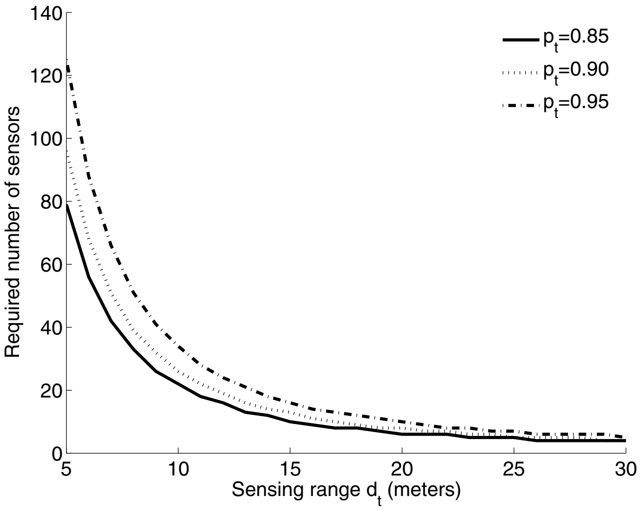

Fig. 3. Effect of sensing range on the required number of sensors for $D_1 = 30$ m, $D_2 = 100$ m and $p_t = 0.85, 0.90, 0.95$.

sensors increases, the resulting DQM performance improves (Fig. 2). Therefore, for denser deployments, the required DQM can be met by sensors with shorter range thereby reducing the power consumption per node. When the sensing range is equal to the width of the field, the DQM is one, and only a couple of randomly deployed sensors are adequate to cover the field as seen in Fig. 3.

Keeping the field area constant at 3000 m^2, the shape parameter, ζ, determined by the dimensions of the rectangle is influential on the required number of sensors as it can be seen in Fig. 4, where the sensing range is 10 m. As the field gets more uniform in shape, fewer sensors are sufficient to deliver the required DQM. As the required deployment quality increases, more sensors are needed. For example, when $\zeta = D_1/D_2 = 0.0083$, 58 sensors are required to provide a deployment quality of $\mathcal{P}_t = 0.85$. However, if $\zeta = 0.30$, deploying 22 sensors is adequate. The effect of the field shape is more influential when the shape parameter is quite small. That is, when the field is a rather narrow and long region, more sensors must be employed. When $\zeta \geq 0.1$, the shape

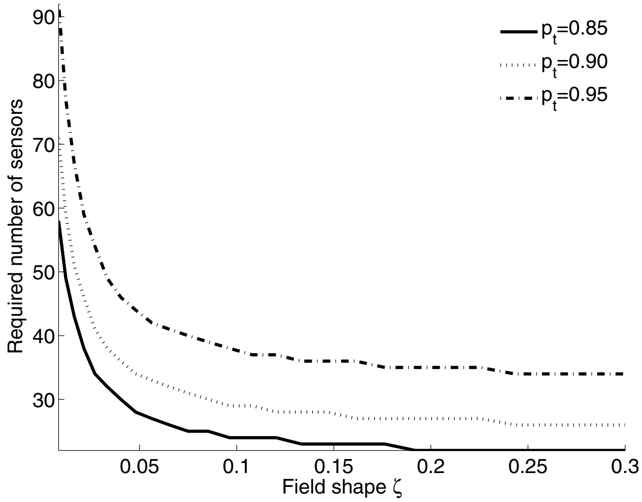

Fig. 4. Effect of field shape parameter $\zeta = D_1/D_2$ on the required number of sensors where total area is 3000 m^2, $d_t = 10$ m. and $\mathcal{P}_t = 0.85, 0.90, 0.95$.

does not affect the required number of sensors much.

The average sensing- and communication-neighbor degrees increase as the sensing range increases. For example, when 10 sensors with a sensing range of 7 m are utilized, the average sensing-neighbor degree is 1.02. The average communication degree is 3.15 if the communication range of the sensor is 14 m. When the network is planned according to the sensing coverage, it can be considered as overengineered in terms of the communication degree. Hence, high redundancy for communication is provided if the network is planned according to sensing, which is the main functionality of the network.

IV. Conclusion

An analytical model to determine the deployment quality is proposed in this paper when sensors are treated as binary detectors. Using this model, it is possible to calculate the required number of sensors to provide the predetermined deployment quality level. Some routing protocols depend on the neighboring degree of sensors. The sensing- and communication-neighbor degrees can be calculated with this model, as well. The analytical evaluation results closely match the simulation outcomes. The designer of the network may use the sensing- and communication-neighbor degrees as decision criteria along with the threshold DQM level.

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