Performance Analysis for OFDMA in the Presence of Tone Interference

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Abstract—Space–frequency coded (SFC) orthogonal frequency-division multiple access (OFDMA) is considered in the presence of multitone (MT) interference. Analytical expressions for the bit-error probability (BEP) are derived for OFDMA with SFC in a frequency-selective fading environment. It is shown that SFC increases the resistance of OFDMA against the MT interference and reduces BEP considerably.

Index Terms—Multitone (MT) interference, orthogonal frequency-division multiple access (OFDMA), space–frequency block coding.

I. INTRODUCTION

In many applications of orthogonal frequency-division multiplexing (OFDM), systems are disturbed by narrowband interference in the channel, which causes an increment in the error probabilities, and hence, a degradation in the quality of communication. For instance, inappropriate spectrum-allocation plans or intermodulation products triggered by bad receiver designs can engender the OFDM system in digital video broadcasting (DVB) to be affected by tone interference [1].

There is a growing interest in employing a multicarrier, multuser system such as orthogonal frequency-division multiple access (OFDMA) for high-rate data transmission in future generation tactical communication systems [2], which must be immune to various forms of intentional narrowband jamming, such as the multitone (MT) variety [3]. Therefore, there is a need for proper analytical tools to assess the performance of OFDMA in the presence of interfering tones. Moreover, appropriate measures need to be taken in order to ensure that interference does not hamper communication.

In this letter, OFDMA with space–frequency coding (SFC) over two transmit antennas is considered, and its performance is evaluated in the presence of MT interference in frequency-selective channels. It is proven that SFC supplies interference-mitigation capability because there are instances where the interference may not affect both subcarriers that are used in OFDMA, and the post-decoding signal-to-interference-plus-noise ratio (SINR) is higher than that experienced by OFDMA alone.

II. SPACE–FREQUENCY BLOCK-CODED OFDMA

The total of \( N \) available subcarriers is distributed over \( U = N/M \) users, each having \( M \) subcarriers which do not need to be contiguous. Data are modulated with quadrature phase-shift keying (QPSK), and fed into a serial-to-parallel converter to generate the OFDM block that is composed of the \( i \)th user’s symbols \( S_i, n = 0, 1, \ldots, M - 1 \). Following space–frequency block coding as in [4], inverse discrete Fourier transformation (IDFT), cyclic prefix (CP) addition, transmission takes place over two antennas which are positioned such that the corresponding channels are uncorrelated.

At the receiver, after CP removal, the discrete Fourier transform (DFT) is applied. The DFTs of channel impulse responses \( H_{i, \ell, n} \) for the \( i \)th user’s transmit antenna \( \ell \) and subcarrier \( n \) are assumed to stay constant for the duration of an OFDM block. Subcarrier \( n \) experiences zero-mean, complex, additive white Gaussian noise (AWGN) with two-sided power spectral density (PSD) of \( N_0/2 \), which is represented by \( Z_n \). (See [5] for more on the OFDMA transceiver.)

Let \( \Lambda \) and \( \Omega \) denote the vector consisting of the channel gains of each user’s \( i \)th transmit antenna, \( \ell = 1, 2 \), and the symbols, respectively. For example, when \( M = 2 \) and \( N = 8 \), \( \Lambda = [H_{1,1,0}, H_{2,2,0}, H_{3,1,0}, H_{4,1,0}]^T \), \( \Lambda_2 = [H_{1,2,0}, H_{2,2,0}, H_{3,2,0}, H_{4,2,0}]^T \) where \( H_{i, \ell, m} := [H_{i, \ell, m} H_{i, \ell, m+1}]^T \), \( \Omega = [S_{1,0} S_{2,0} S_{3,0} S_{4,0}]^T \), and \( S_{i, m} := [S_{i, m} S_{i, m+1}]^T \), for \( m = 0, 2, 4, \ldots, M - 2 \).

Assuming the availability of perfect channel state information (CSI) at the receiver, we adopt the zero-forcing space–frequency decoder of Vielmon et al. [6], which preserves the variance of noise or interference as they are at the input. Consequently, the SFC decoder outputs are

\[
\hat{\Omega}_m = \frac{[\Lambda_{1,m} \Lambda_{2,m+1}] + [\Lambda_2, m \Lambda_2, m+1]}{\sqrt{[\Lambda_{1,m}^2 + [\Lambda_2, m]^2]} \sqrt{\hat{\Omega}_m}}
\]

\[
\hat{\Omega}_m+1 = \frac{[\Lambda_{1,m} \Lambda_{2,m+1}] + [\Lambda_2, m \Lambda_2, m+1]}{\sqrt{[\Lambda_{1,m}^2 + [\Lambda_2, m]^2]} \sqrt{\hat{\Omega}_m}}
\]

for \( m = 0, 2, 4, \ldots, N - 2 \), where \( \Lambda_{i,m} \) and \( \Omega_m \) are the \( m \)th elements of \( \Lambda \) and \( \Omega \), respectively; \( \hat{Z}_m \) and \( Z_{m+1} \) are complex white Gaussian with the same mean and variance as \( Z_m \) and \( Z_{m+1} \) when conditioned on the channel gains. Suppose that \( \Lambda_{1,m+1} \approx \Lambda_{1,m} \) and \( \Lambda_{2,m+1} \approx \Lambda_{2,m} \) (see [5] for justification). The SINR of the \( m \)th subcarrier at the decoder output when interference is absent (denoted by \( \gamma_m^0 \)) is represented as

\[
\gamma_m^0 = \left( [\Lambda_{1,m}^2 + [\Lambda_2, m]^2] \right) \gamma S \approx \gamma_{m+1}^0
\]
where $\gamma_S = E_s / N_0$ with $E_s$ denoting the symbol energy. The random variable $\gamma_m^2$ has a chi-squared probability density function (pdf) with four degrees of freedom for all $m$, which is $p(\gamma_m^2) = \frac{\gamma_m^2 e^{-\gamma_m^2/4}}{2^{d-1} \Gamma(d/2)}$.

The total MT interference power $P_{\text{tot}}$ is distributed over $T$ contiguous or noncontiguous subbands so that $\alpha = T / N$ of the bandwidth is affected by the tones. Assuming that $P_{\text{tot}}$ is uniformly distributed over the entire bandwidth $W_{\text{tot}}$, the effective interference PSD in any subband is $P_t = P_{\text{tot}} / W_{\text{tot}}$.

Let $I$ be the index set of subcarriers hit by equal-power, contiguous or noncontiguous placed tones for some OFDM block. The interference source does not commit to specific subbands, and the users do not have prior information on $I$. The MT interference is modeled as $I(t) = \sum_{k \in I} \sqrt{P_t} e^{j(2\pi f_k t + \phi_k)}$ where $P = P_{\text{tot}} / T$ is the average power of each tone, and $f_k$ and $\phi_k$ are the frequency and phase of the tone that hits the $k$th subcarrier, respectively. To consider the effects of MT interference on the SINR at the SF decoder output, three distinct hit scenarios are examined in terms of the subcarrier pairs involved in SFC.

(S1) Both subcarriers are disturbed by the tones.

The received symbols are observed as

$$Y_m = A_{1,m} \Omega_m + A_{2,m} \Omega_{m+1} + G_{m} I_m + Z_m$$

$$Y_{m+1}^* = -A_{1,m+1}^* \Omega_m + A_{2,m+1}^* \Omega_{m+1} + G_{m+1}^* I_{m+1}^* + Z_{m+1}^*$$

and the corresponding SINRs at the decoder output become

$$\gamma_{m+1} = \frac{[\gamma_{m+1}^2 + |A_{2,m}^2| E_s]}{N_0 + (|G_m|^2 + |G_{m+1}|^2) P_t} \approx \gamma_{m+1}^2$$

where $I_m$ and $G_m$ represent the interference tone and the channel transfer function experienced by the interference, respectively, and $P_t = P_t / \alpha$.

(S2) A single subcarrier is disturbed by the tones.

The resulting SINRs, $\gamma_m^2 \approx \gamma_{m+1}^2$, are determined by setting $G_m = 0$ or $G_{m+1} = 0$ in (2).

(S3) None of the subcarriers is disturbed by the tones.

This scenario is already investigated in (1).

III. COMPUTATION OF THE TONE-INTERFERENCE PROBABILITIES FOR SFC

In this section, the hit probabilities are derived with the assumption that tones are placed noncontiguously; the case of contiguous tones is considered in [5]. While the tones are generated simultaneously, we treat them as if they were ordered in time for the sake of convenience in the derivations below.

A. Probability That a Subcarrier Pair Experiences a Double Tone Interference

Let $D_k$ represent the event that any subcarrier pair is double-hit, given that $k$ out of $M/2$ pairs are also double-hit. Then, the probability $P(D_k) = 2 / (N(N - 1))$, because once a subcarrier is hit with probability (w.p.) $2 / N$, there are $N - 1$ remaining subcarriers equally likely to be interfered and tones may be placed noncontiguously. Likewise, $P(D_k) = 2 / (N - 2)(N - 3)$ because now the number of subcarriers that can be potentially interfered is $N - 2$, each w.p. $1 / (N - 2)$. After the first hit, the probability that the other subcarrier of the pair is also hit becomes $1 / (N - 3)$. The factor of two accounts for the fact that either of the two subcarriers in a pair may be hit first. Continuing in this fashion, $P(D_k) = 2 / (N - 2k)(N - 2k - 1)$. The probability that $d$ out of $M/2$ subcarrier pairs experience double interference with no constraint on the rest of the pairs, $P_{D,d}(d)$, is

$$p_{D,d}(d) = \frac{M!}{d!} \sum_{k=0}^{d-1} \binom{d}{k} \frac{2}{(N - 2k)(N - 2k - 1)}$$

where $p_{D,d}(0) := 1$.

Given that there are $d$ double-interference pairs, the remaining $M/2 - d$ pairs (i.e., $M - 2d$ subcarriers) are interference-free or subject to single-tone-interference. In total, there are $N - 2d$ subcarriers for the tones to disturb. Let $S_{d,n}$ be the event that any pair has just one of its subcarriers hit, given that $d$ out of $M/2$ pairs are double-interfered and $n$ are single-hit. Then, $P(S_{d,n}) = (M - 2d)/(N - 2d)$. Once it is given that a subcarrier is hit, there remain $M - 2d - 2$ (because the pair has to be single-hit, we subtract two) subcarriers out of $N - 2d - 1$. Continuing, one arrives at $P(S_{d,n}) = (M - 2d - 2n)/(N - 2d - n)$. Letting $p_{S,d}(s|d)$ denote the probability of observing $s$ single-interference pairs, given that there are $d$ double-interference pairs

$$p_{S,d}(s|d) = \prod_{n=0}^{s-1} P(S_{d,n}) = \prod_{n=0}^{s-1} \frac{M - 2(d + n)}{N - 2d - n}.$$
yields the probability that \(d\) pairs are subject to double-interference as

\[
p_D(d) = p_{D,0}(d) \sum_{s=0}^{\min(M/2,T/2-d)} \frac{T!}{2^s s! (T-2d-s)!} \gamma_S(s|d) P(F_{d,s}).
\]

The expected number of subcarrier pairs subject to a double tone interference is

\[
\bar{d} = \sum_{d=1}^{\min(M/2,T/2)} d \cdot p_D(d).
\]

Finally, the probability that any subcarrier pair experiences a double tone interference is

\[
P_2 = \frac{\bar{d}}{M^2}.
\]

### B. Probability That a Subcarrier Pair Experiences a Single Tone Interference

The derivation here is similar to the double-tone-interference case above. Considering the tone-ordering mechanism, with each tone arrival, the subcarriers that can be hit next decrease in pairs, whereas the tones themselves decrease one by one. Thus, the probability that \(s\) out of \(M/2\) subcarrier pairs experience single-interference with no constraint on the rest of the pairs is \(\prod_{k=0}^{s-1} (M - 2k)/(N - k)\). The remaining \(M/2 - s\) pairs are interference-free or subject to double-interference. \((M/2-s)\prod_{m=0}^{T/2-s-1} (N - M - m)/(N - 2d - s - m)\) is the probability of observing \(d\) double-interference pairs given that there are \(s\) single-interference pairs, where \(d\) ranges from 0 to \([\min(M/2 - s, (T - s)/2)]\). Next, the number of interference-free pairs is \(M/2 - d - s\), and \(\prod_{m=0}^{T-2d-s-1} (N - M - m)/(N - 2d - s - m)\) corresponds to its occurrence probability. Putting all the above calculations together and recalling that there are \(T!(2^d s! (T-2d-s)!\) interference scenarios, the probability that \(s\) pairs are subject to single-interference is shown in the equation at the bottom of the page.

The expected number of carrier pairs subject to a single tone-interference is

\[
\bar{s} = \sum_{s=1}^{\min(M/2,T)} s \cdot p_S(s),
\]

and consequently, any given subcarrier pair experiences a single tone interference w.p. \(P_1 = \bar{s}/(M/2)\).

### IV. Bit-Error Probability (BEP) Performance Under MT Interference

Each user transmits over \(M\) subcarriers which are assigned such that collisions are avoided. For simplification, the following assumption is taken into account.

(A1) The interferer signal is assumed not to experience fading at any subband, and therefore \(|G_k|^2 = 1, k = 0,1,\ldots, N-1\). Simulations where the interferer does go through independent fading show that there might be over two orders of magnitude BEP enhancement at signal-to-tone ratio \(\text{STR} := E_b/\nu = 20\) dB for channels with severe nulls. However, the average BEP over 1000 channel realizations is marginally better than the case in (A1), because in general, the interferer’s band does not fully overlap with the null, particularly if \(\alpha\) is small.

The expected BEP of SFC-OFDMA over a Rayleigh fading channel is

\[
P_b^{\text{SFC-OFDMA}} = \frac{1}{N} \int_0^\infty \cdots \int_0^\infty \left( P_0^0 + P_0^2 + \cdots + P_0^{N-2} \right) \times p(\Delta_1) p(\Delta_2) d\Delta_1 d\Delta_2
\]

where \(\Delta_\ell = \gamma_S[|\Delta_{\ell,0}|^2 \cdots |\Delta_{\ell,N-1}|^2]^T, p(\Delta_\ell)\) is the multi-variate chi-squared pdf for the channel gains experienced by the \(\ell\)th transmit antennas of all users, \(\ell = 1, 2,\) and

\[
P_{b,m,0} = P_2(P_{b,m,2} + P_{b,m+1,2}) + P_1(P_{b,m,1} + P_{b,m+1,1}) + P_0(P_{b,m,0} + P_{b,m+1,0})
\]

where \(P_1, P_2\) are defined in Section III. \(P_0 = 1 - P_1 - P_2,\) and \(P_{b,m,0}\) is the BEP for the given SINR in the \(m\)th subcarrier when \(i\) subcarriers of the pair \((m, m+1)\) are hit, with [3]

\[
P_{b,m,0} = \mathcal{Q}\left(\sqrt{\frac{E_b}{2N_0}}\right) \approx P_{b,m,1,0}
\]

\[
P_{b,m,1} = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{Q}\left(\sqrt{\frac{E_b}{2N_0}} \sin \theta \right) d\theta \approx P_{b,m,1,1}
\]

\[
P_{b,m,2} = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{Q}\left(\sqrt{\frac{E_b}{2N_0}} \sin \theta \right) d\theta \approx P_{b,m,1,2}
\]

and \(\mathcal{Q}(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-y^2/2} dy\).

Define \(P_b^{\text{SFC-OFDMA}} = P_0^0 + P_0^2 + \cdots + P_0^{N-2}\), so that \(P_b^{\text{SFC-OFDMA}} = P_0^0 + P_0^2 + \cdots + P_0^{N-2}\), and make the following assumption next.

(A2) Per user, the channel gains of the SFC subcarrier pairs are mutually statistically independent. This is particularly justifiable when the pairs are not contiguous, and because only \(M < N\) subcarriers are assigned to each terminal. Moreover,
distinct OFDMA users communicate over uncorrelated channels in the uplink.

By (A2), each $2N$-tuple integral in (3) becomes

$$P_b^m = \frac{1}{N} \int_0^\infty \cdots \int_0^\infty P_b^m \rho(\Delta_{1,m}) \rho(\Delta_{2,m}) d\Delta_{1,m} d\Delta_{2,m}$$

where $\Delta_{\ell,m} = \frac{\gamma_m [\Lambda_{\ell,m}]^2 \Lambda_{\ell,m+1}^2}{N_0}$ and $\rho(\Delta_{\ell,m})$ is the associated bivariate chi-squared pdf, $\ell = 1, 2$.

All $P_b^m$ are identically distributed. Recalling the assumption in Section II that adjacent subcarriers have approximately the same channel gains, and concentrating on $P_b^0$, the expected BEP expression in (3) reduces to

$$P_b^{SFC-OFDMA} = \frac{N}{2} P_0^0 \left[ 2 \pi \int_0^\infty \int_0^\infty Q \left( \sqrt{\frac{\gamma_0^0}{N_0}} - \sqrt{\frac{4P_s}{N_0}} \sin \theta \right) \rho(\gamma_0^0) d\gamma_0^0 d\theta ight]$$

These integrals can be numerically evaluated by using the trapezoidal integral evaluation method and the Gauss–Laguerre integration formula [7]. For fixed $E_b/N_0$ and $E_b/P_s$, if the number of disturbed carriers increases, it will lead $P_2$ to increase, whereas $P_0$ will decrease. On the other hand, because increasing $\alpha$ reduces $P_1$, there is a tradeoff between $\alpha$ and $P_1$.

When tones are placed contiguously, they cause the double tone interferences to occur more frequently than single tone interferences, because the contiguous interference scenario prevents the expected number of single-tone-interference pairs to exceed two [5]. Thus, double-tone-interference occurrences become more dominant. However, when noncontiguous interference is considered, the single-tone-interference occurrences increase, which causes a reduction in the double-tone-interference occurrences. It can be concluded that contiguous tones are more harmful to communication than the noncontiguous ones.

V. NUMERICAL EVALUATIONS AND SIMULATIONS

There are $N = 256$ available subcarriers in total and $M = 8$ to supply 32 users. The OFDMA simulations are performed with a single transmit/receive antenna pair, whereas there are two transmit and one receive antennas in SFC-OFDMA simulations. The total transmitted power per subcarrier is set to $E_b$ in both cases (see [8] for performance comparison when this power constraint is not enforced). The position of the interference tones randomly changes at every OFDM block. Symbols with QPSK modulation are transmitted over frequency-selective channels modeled as finite impulse response (FIR) filters that are of order five with complex Gaussian-distributed coefficients and flat power delay profile, and contaminated by zero-mean, complex AWGN. The channels experienced by each antenna are assumed to be uncorrelated, and CSI is perfectly estimated at the receiver. The CP length is set to six symbols. The simulation results reflect the averages of 1000 independent channel pair realizations. In the graphs, we use the SNR definition of $SNR = E_b/N_0$.

Fig. 1 depicts that at high SNRs, the performance-limiting factor is the MT interference. At high STR and SNR, the diversity gain offered by SFC becomes noticeable. For a BEP of $1 \times 10^{-3}$, the OFDMA system with SFC requires about 6 dB less energy to combat noise and interference, compared with OFDMA alone at $STR = 20$ dB. Fig. 2 shows the BEP performance against STR for $SNR = 20$ dB. As the STR increases, the BEP performance is augmented and SFC supplies an 8-dB STR improvement at a BEP of $5 \times 10^{-3}$ for $SNR = 20$ dB. It should be noted that at low STRs, double-hit error probabilities increase enormously, and therefore SFC-OFDMA performs slightly worse than OFDMA.
Fig. 3 shows the BEP performance comparison of the noncontiguous and contiguous tone scenarios against STR and SNR. Notice that contiguous placement of tones produces the most damage for all STR values. The difference arises from the fact that in the contiguous tone case, the dominant interference type is double-tone, whereas with noncontiguous tones, single-tone interference dominates (e.g., for $\alpha = 0.5$ and noncontiguous tones, 0.5080, 0.2460, 0.2460 are the probabilities with which a subcarrier pair experiences single-tone interference, double-tone interference, or interference-free, respectively, while they are 0.0304, 0.4848, 0.4848 for contiguous tones).

REFERENCES


